# Vehicle Dynamics and Simulation

## Drivetrain Dynamics

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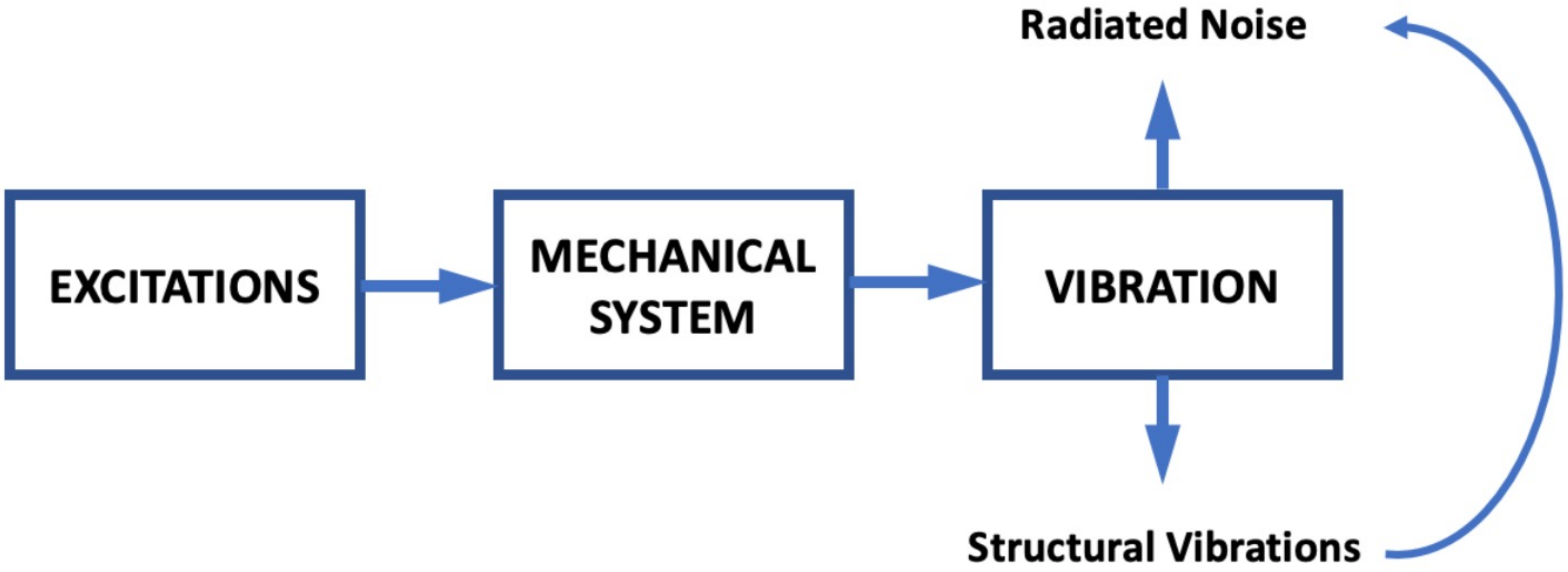
# Note

- The test is on Tuesday at 2pm

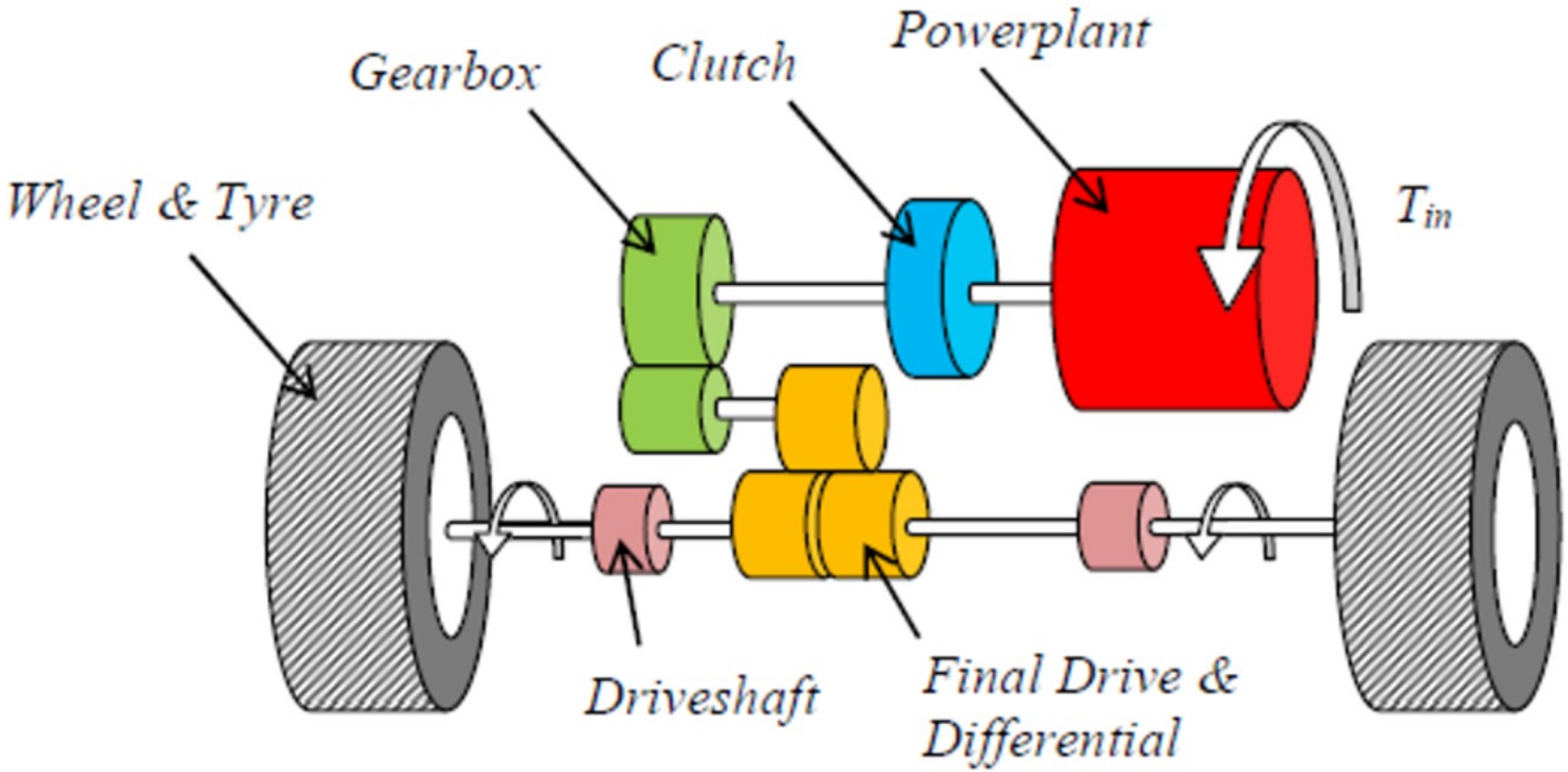
# Lecture Overview

- Drivetrain as a vibrational system
- Torsional drivetrain model
- Excitation sources
- Driveline Components
- Vibration analysis

# The Drivetrain System

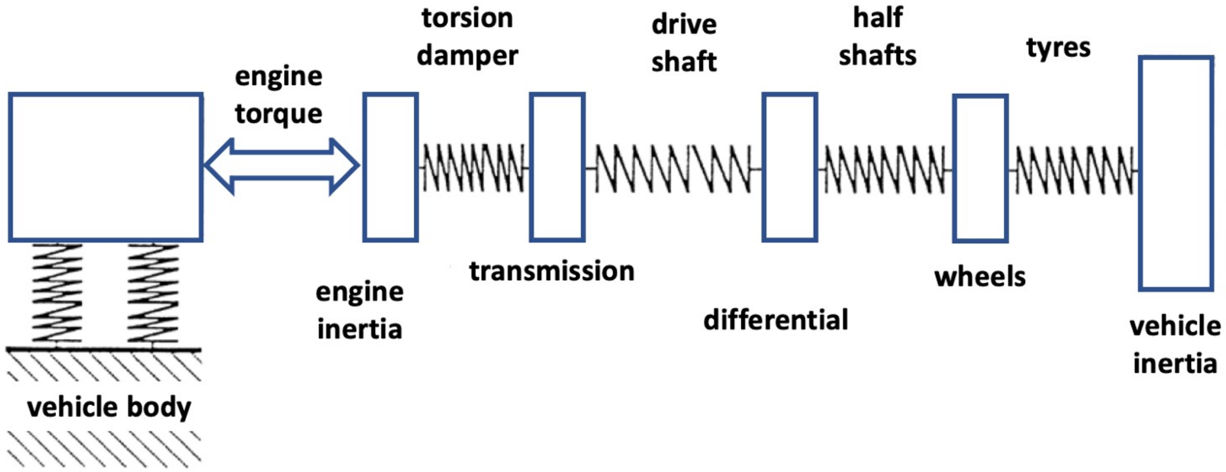


# The Drivetrain System



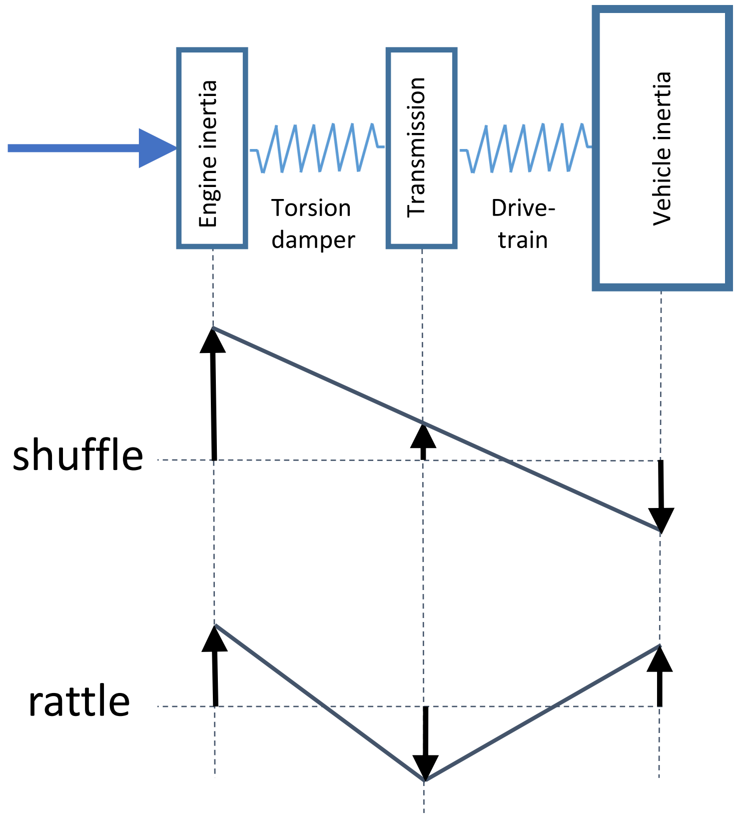
# The Drivetrain System

- The drivetrain can be represented as a number of springs and masses
- Each mass is isolated as a point-mass
- These types of models are known as lumped parameter models
- Control of vibration is achieved by;
  - Reducing excitation
  - Changing stiffness and damping (resonance frequencies)

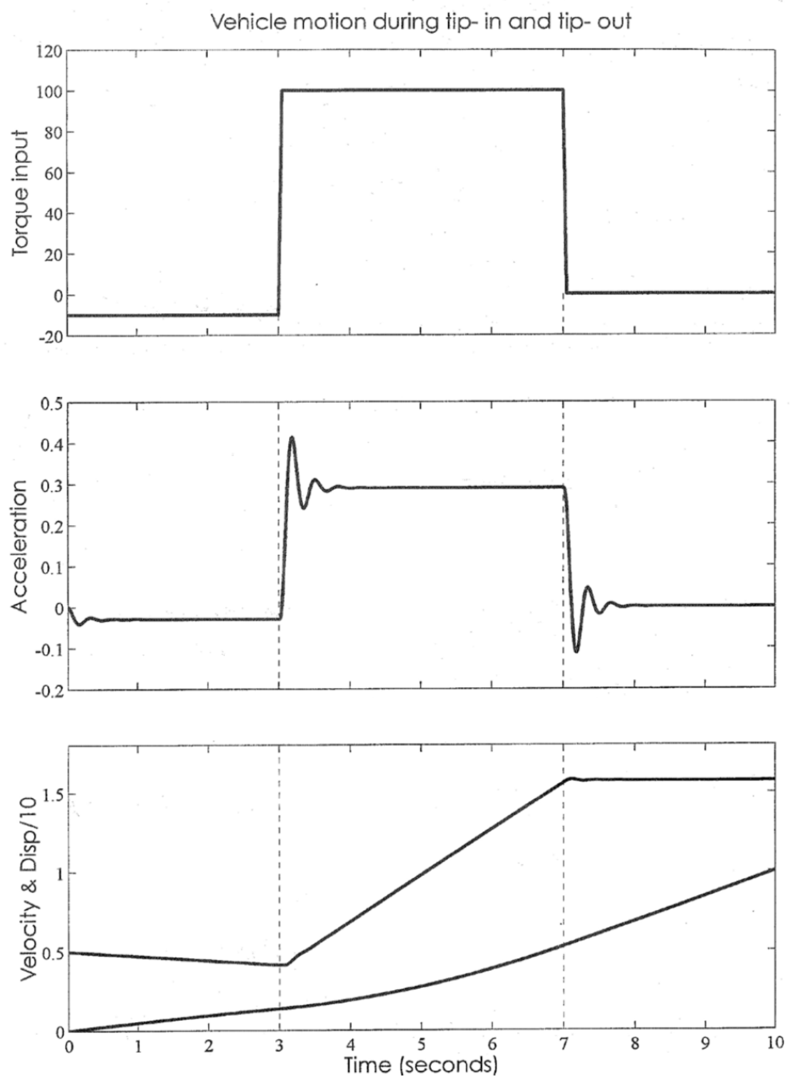


# The simplest useful model

- Three main modes;
  - Shuffle (4-12 Hz)
  - Rattle (40 – 80 Hz)
  - Rigid body rotation
- And others;
  - Boom (interior compartment)
  - Judder (low frequency on clutch engagement)
  - Clonk (lash in driveline)

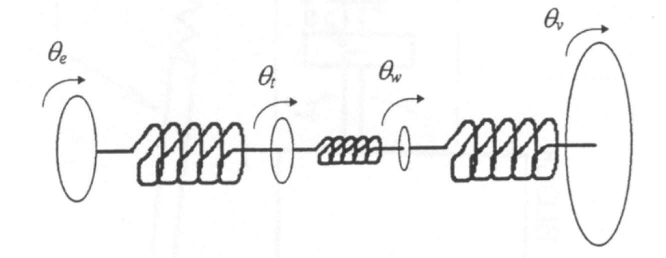


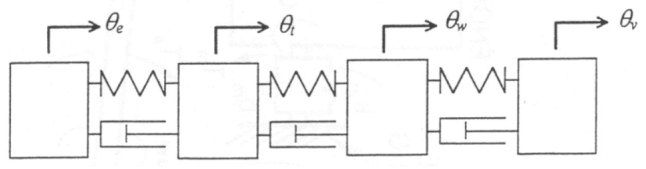
# Response to step input

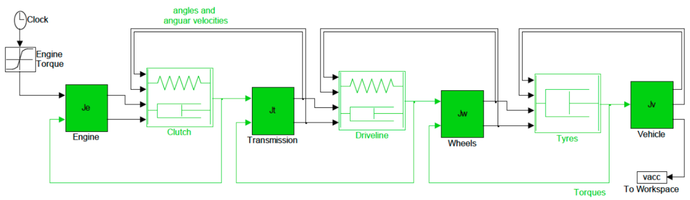


# More advanced models

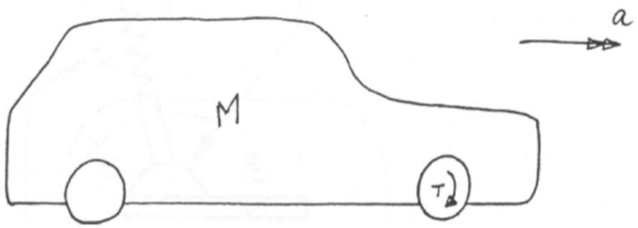
- Three, four, six, …, twenty! mass models
    - Complexity driven by requirements
- Can add masses to suit
- Other components in the drivetrain are important
    - Clutch
    - Differential
    - Actuators and related controls

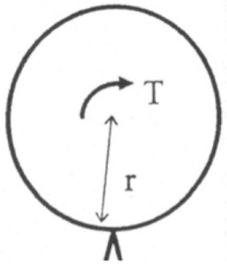


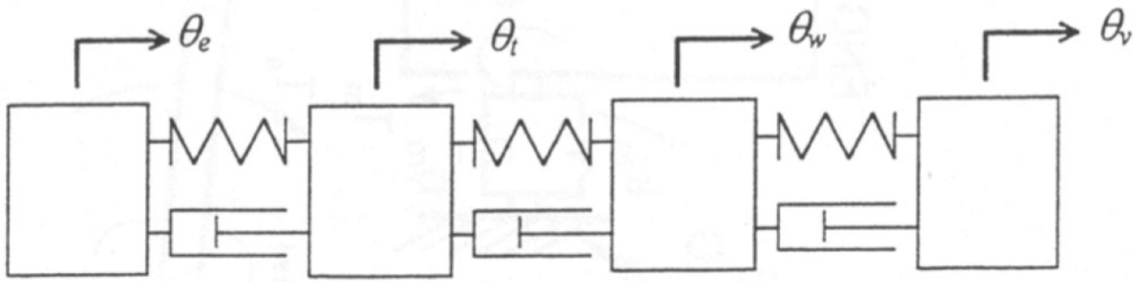



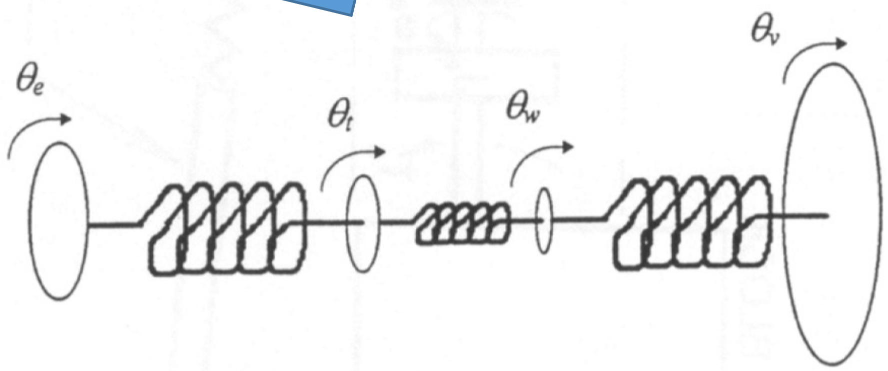



# Rotation or Translation?

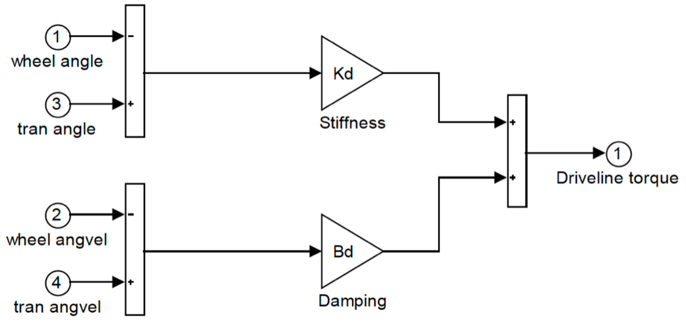


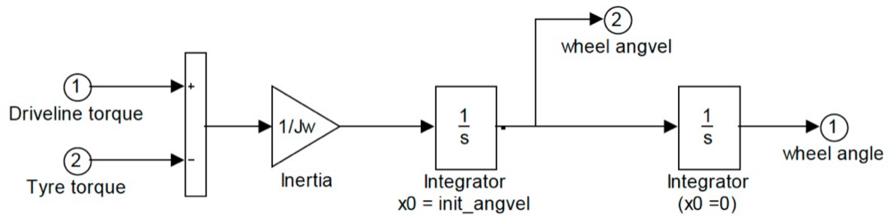



# Standardised (Simulink) components




Spring-damper




Mass

# Sources of excitation

- Main sources of excitation
  - Combustion
  - Step-in and step-out
  - Electric machine
  - HEV transitions
  - Cylinder deactivation
- Transmission
  - Gear meshing
  - Ratio changes
- Driveshaft
  - Universal joints
- Tyres and wheels
  - Runout
  - Mass imbalance
  - Stiffness variations

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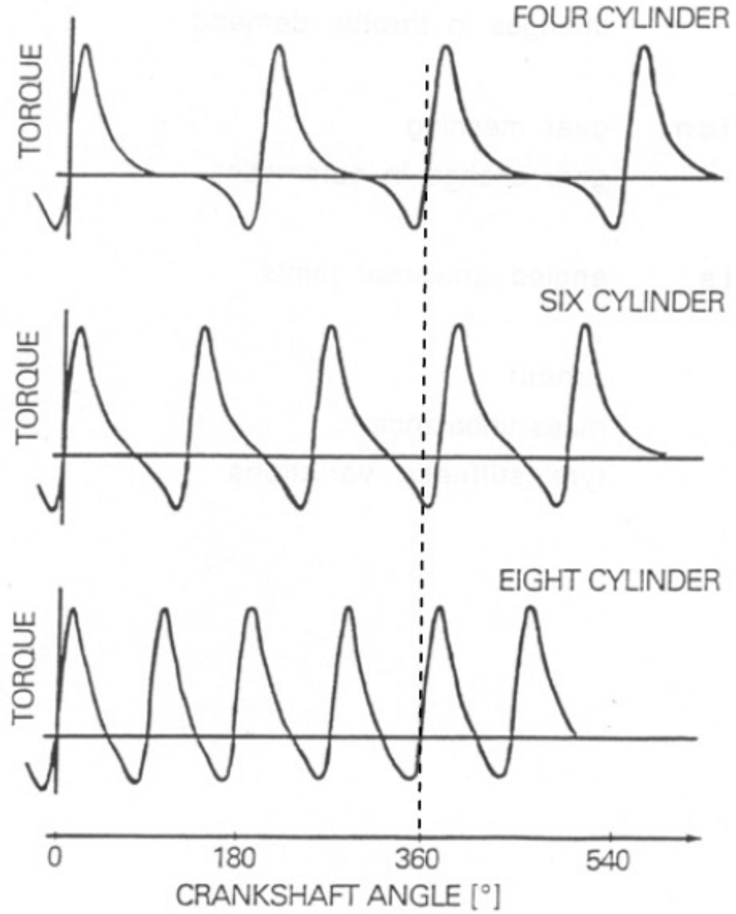
# Excitation - Combustion

- Firing of engine causes torque pulses
- Varies in frequency
  - Number of cylinders
  - Engine speed
- Other events
  - Valve opening and closing (x1 per rotation)
  - Injector opening and closing (x1 per cycle)

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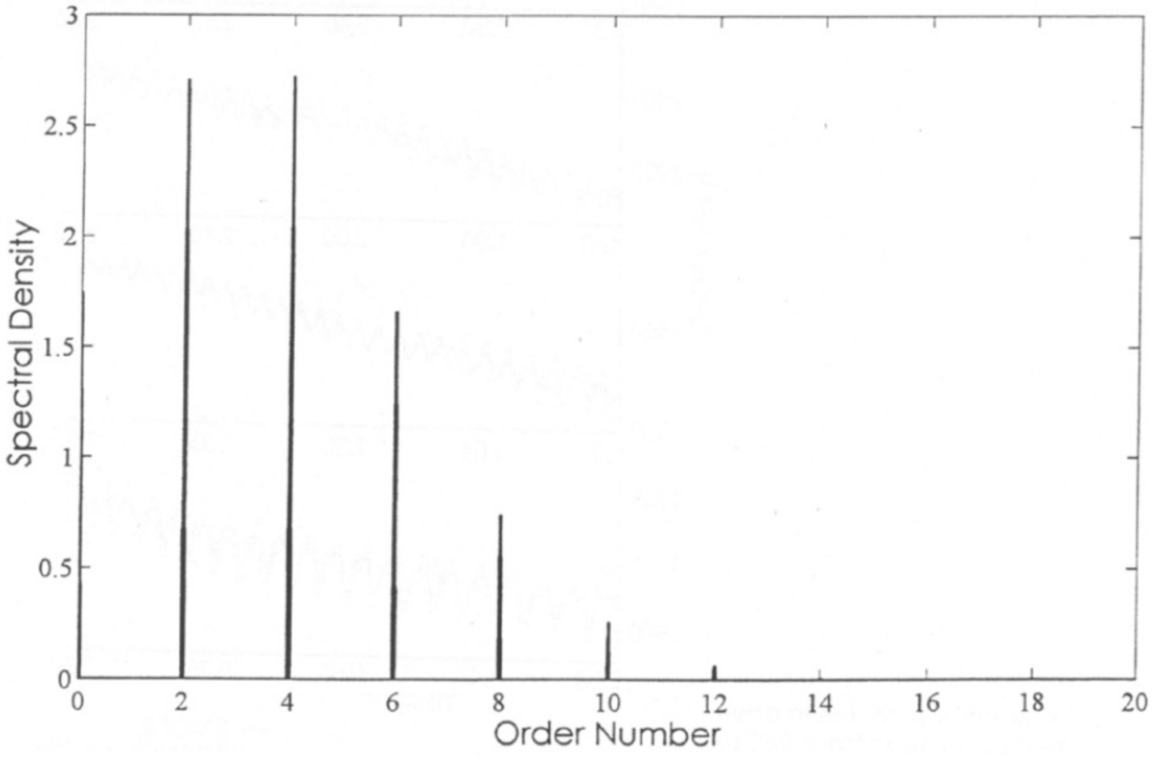
# Excitation - Combustion

- Spectral analysis (FFT) to look at frequency content of measured response
- Can normalise spectral analysis result with rotational speed
  - Shows how response relates to engine speed
- Can also create interaction plot / Campbell diagram to show response relative to excitation (engine).

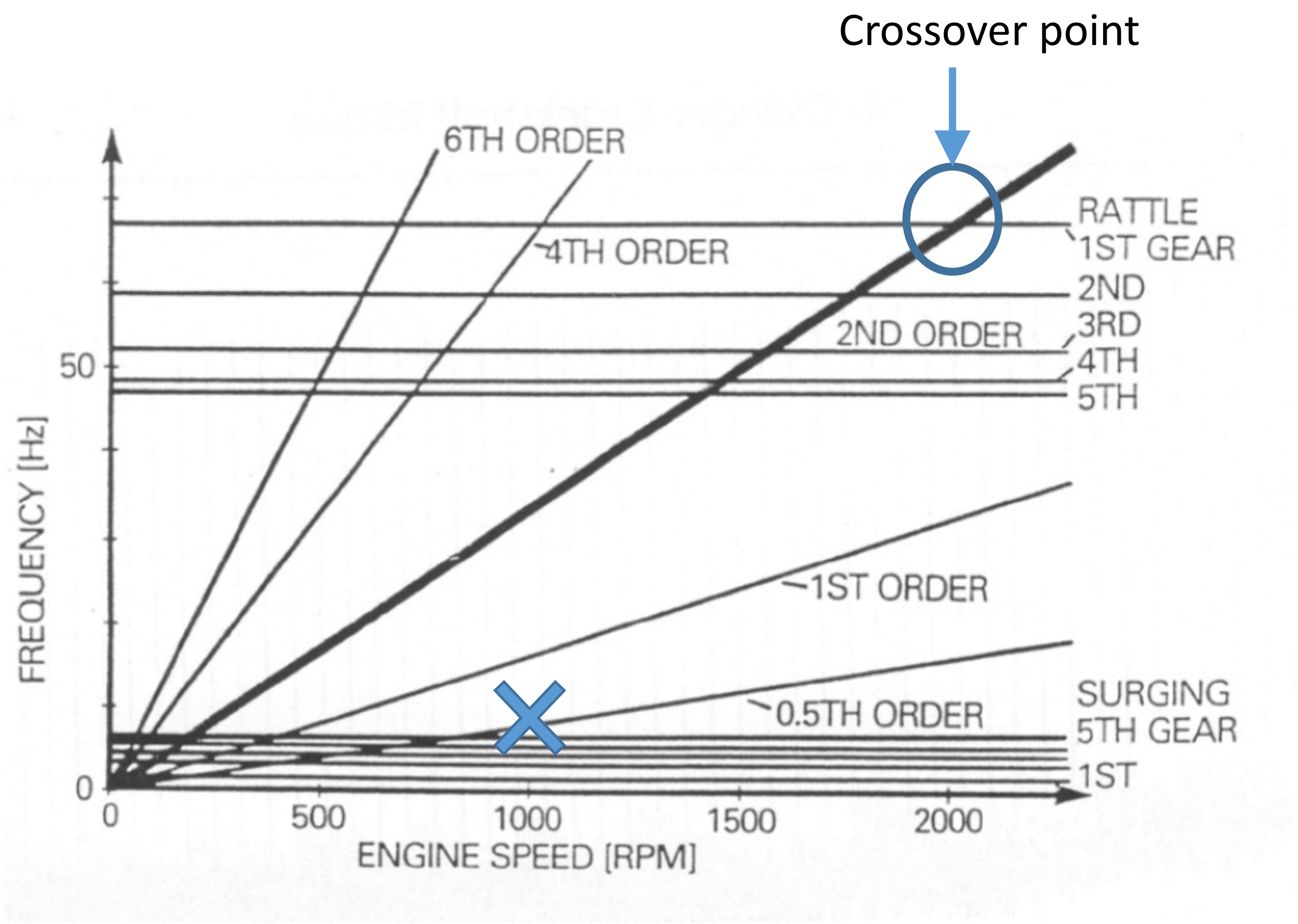
Engine order vibration




Each peak in the diagram above represents a frequency described in terms of the order i.e. frequency/rotational frequency

# Campbell Diagram

- Order is frequency / rotational frequency
- 1st order is 1:1 mapping e.g. at 1000 rpm = 1000 / 60 cycles per second (Hz).
- Where the 'order' crosses the 'resonance' line is the point of max vibration.

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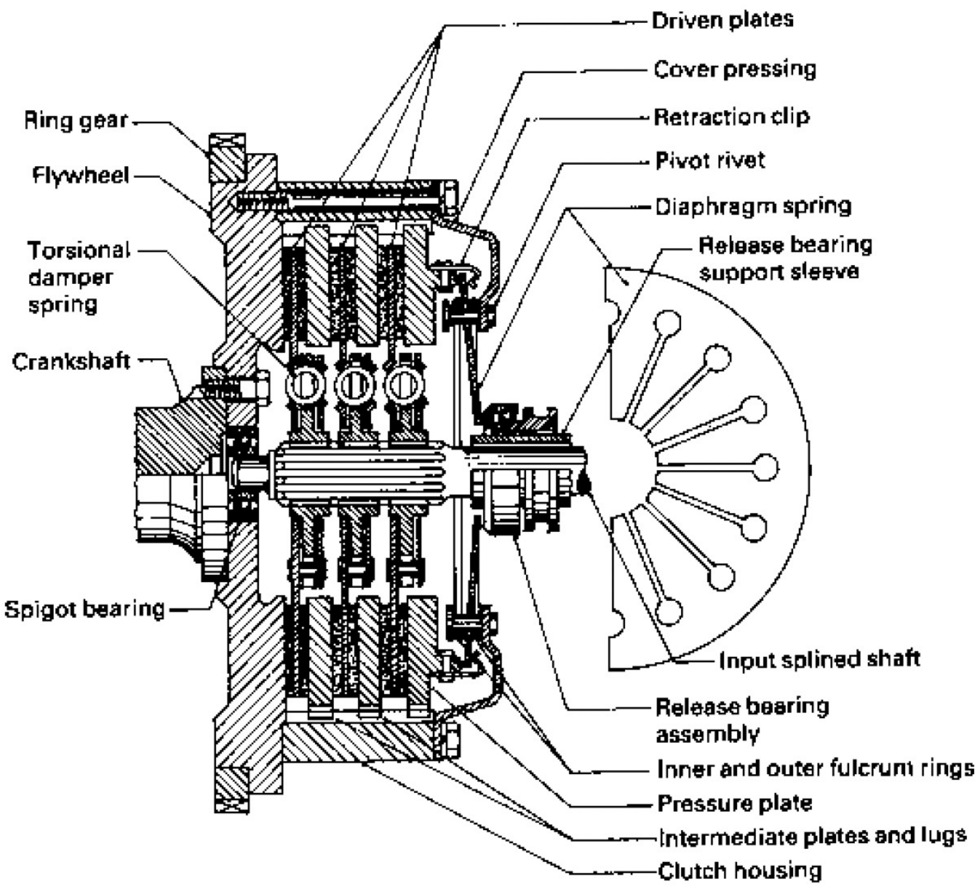
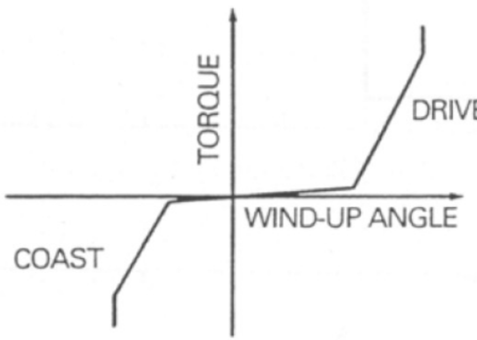
# Torsion Damper

- Used as first line of defence against excitation in the driveline.
- Different physical arrangements.
- Nonlinear spring rate with some damping.



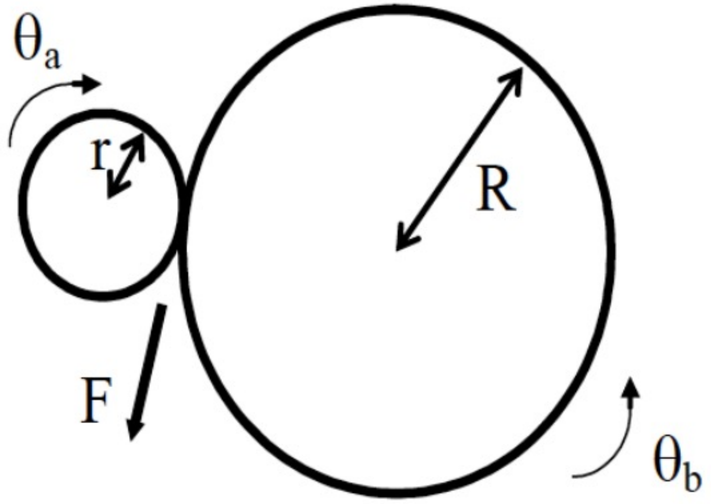
# Gearbox

- Can be modelled as a single pair (and 'switched').
- Output torque is calculated ($T_a$ is input torque);

$$T_b = GT_a$$

- With the gear ratio, G given by;

$$G = R/r$$

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# Simple Drivetrain Model

- Three inertias
- Compliantly connected by rotational springs
- Engine inertia accel;

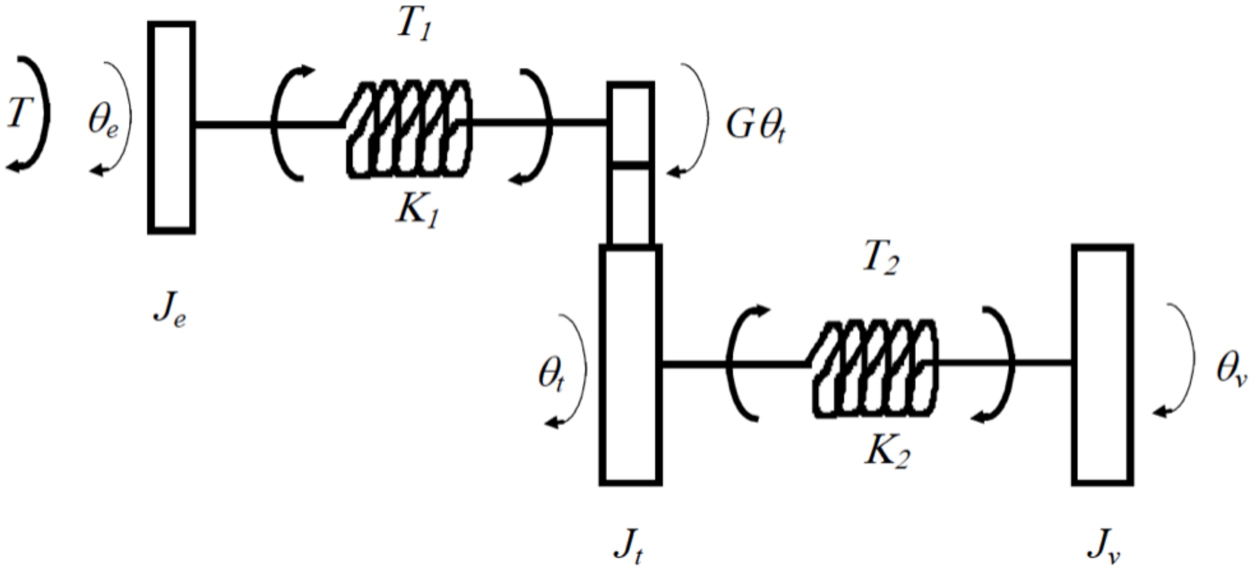
$$J_e\ddot{\theta}_e = T - T_1$$

- Transmission inertia accel;

$$J_t\ddot{\theta}_t = GT_1 - T_2$$

- Vehicle inertia accel;

$$J_v\ddot{\theta}_v = T_2$$

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# Simple Drivetrain Model

- Torque transmitted between engine and transmission;

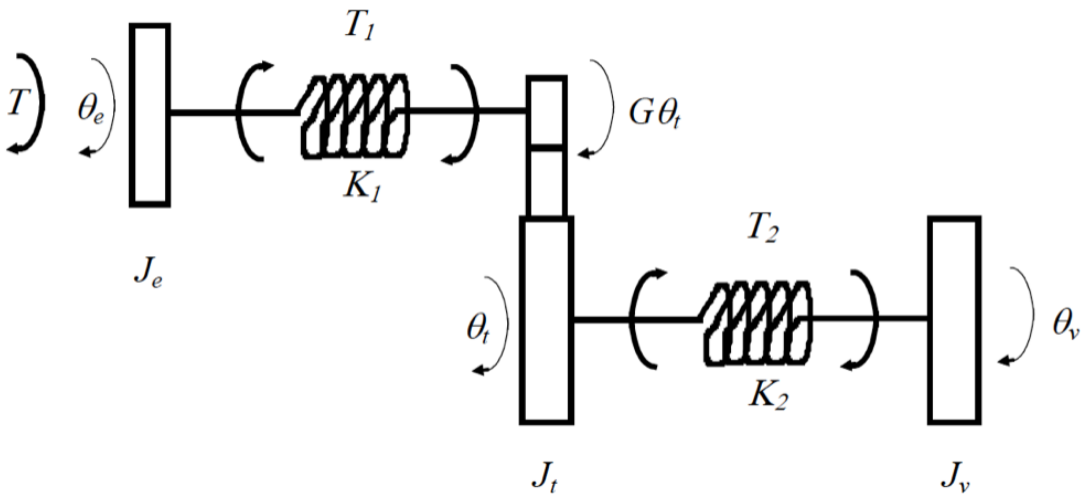
$$T_1 = K_1(\theta_e - G\theta_t)$$

- Torque transmitted between transmission and vehicle;

$$T_2 = K_2(\theta_t - \theta_v)$$

- So that;

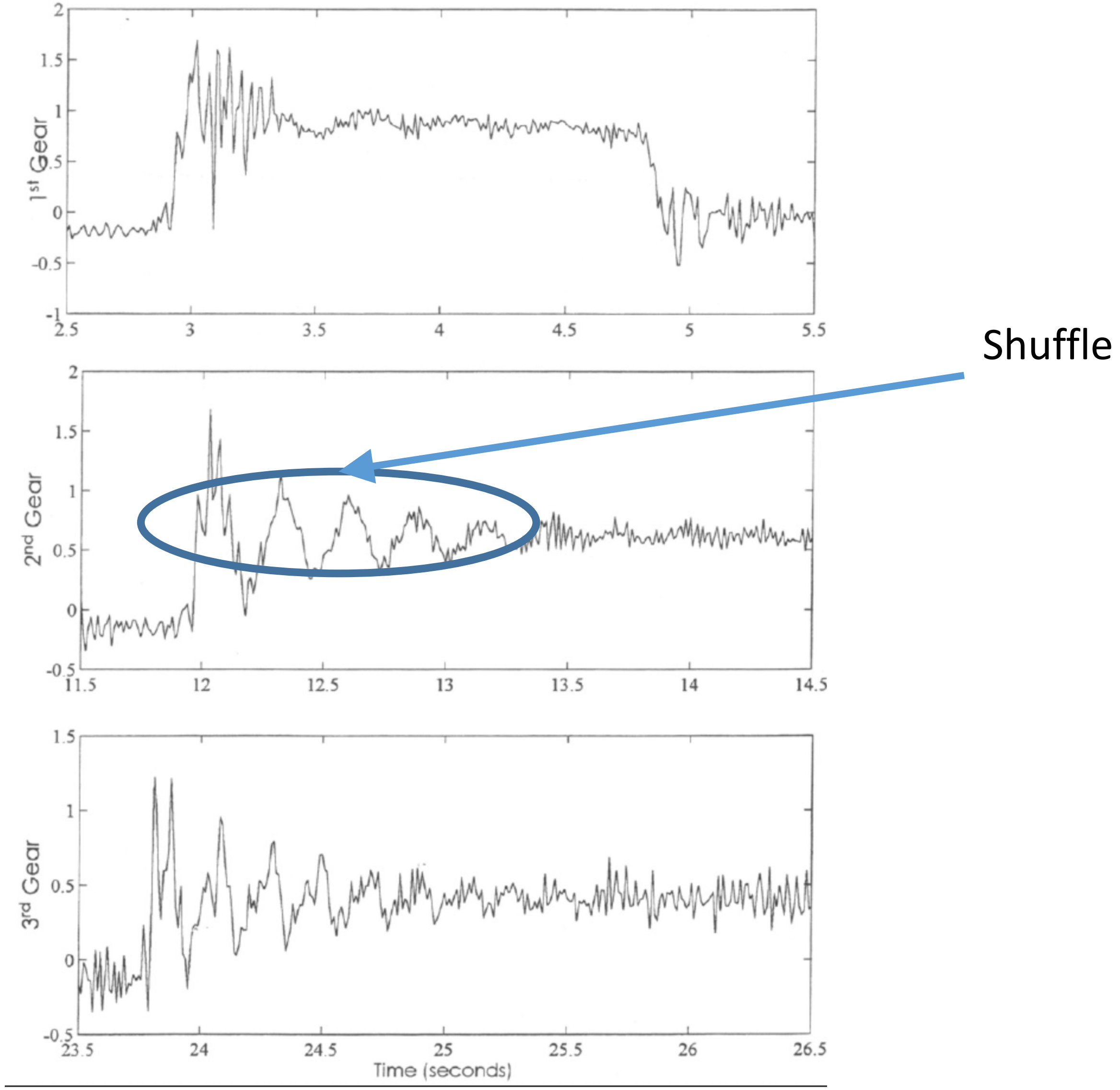
$$J_e\ddot{\theta}_e = T - K_1\left(\theta_e - G\theta_t\right)$$
$$J_t\ddot{\theta}_t = GK_1\left(\theta_e - G\theta_t\right) - K_2\left(\theta_t - \theta_v\right)$$
$$J_v\ddot{\theta}_v = K_2\left(\theta_t - \theta_v\right)$$




- A less messy formulation is (see notes for derivation);

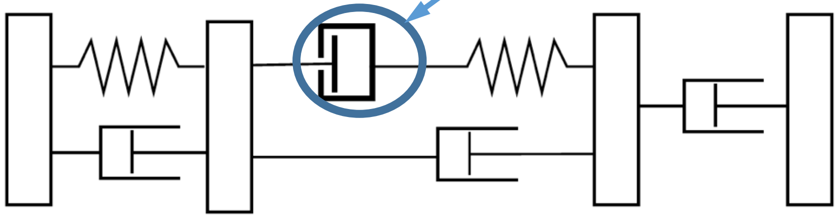
$$J_e^*\ddot{\theta}_e^* = GT - K_1^*\left(\theta_e^* - \theta_t\right)$$
$$J_t\ddot{\theta}_t = K_1\left(\theta_e^* - \theta_t\right) - K_2\left(\theta_t - \theta_v\right)$$
$$J_v\ddot{\theta}_v = K_2\left(\theta_t - \theta_v\right)$$

# Measured Response (Mondeo 1.8l)

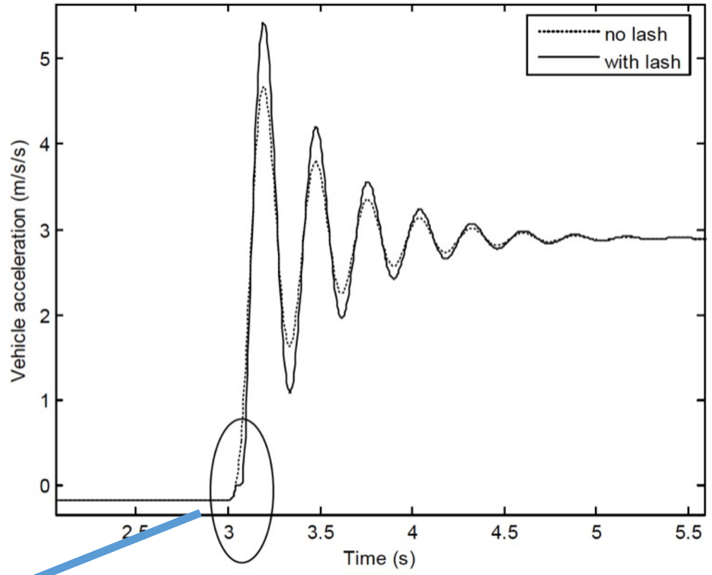


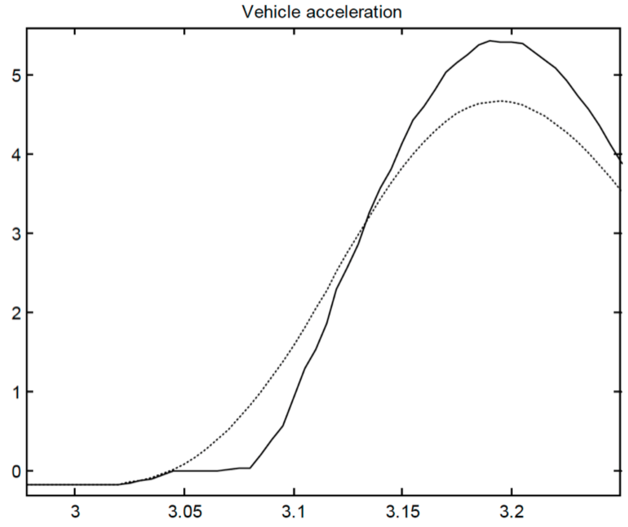
# Driveline Nonlinearities

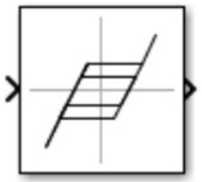
- There are many sources of nonlinearity.
- Typically include 'lash' as most significant.
- Lash element available in Simulink

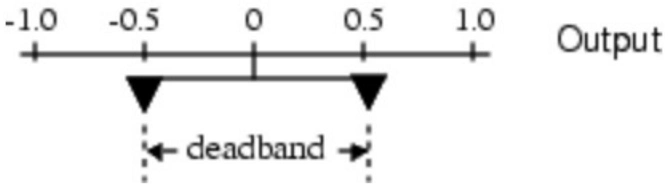



Step input



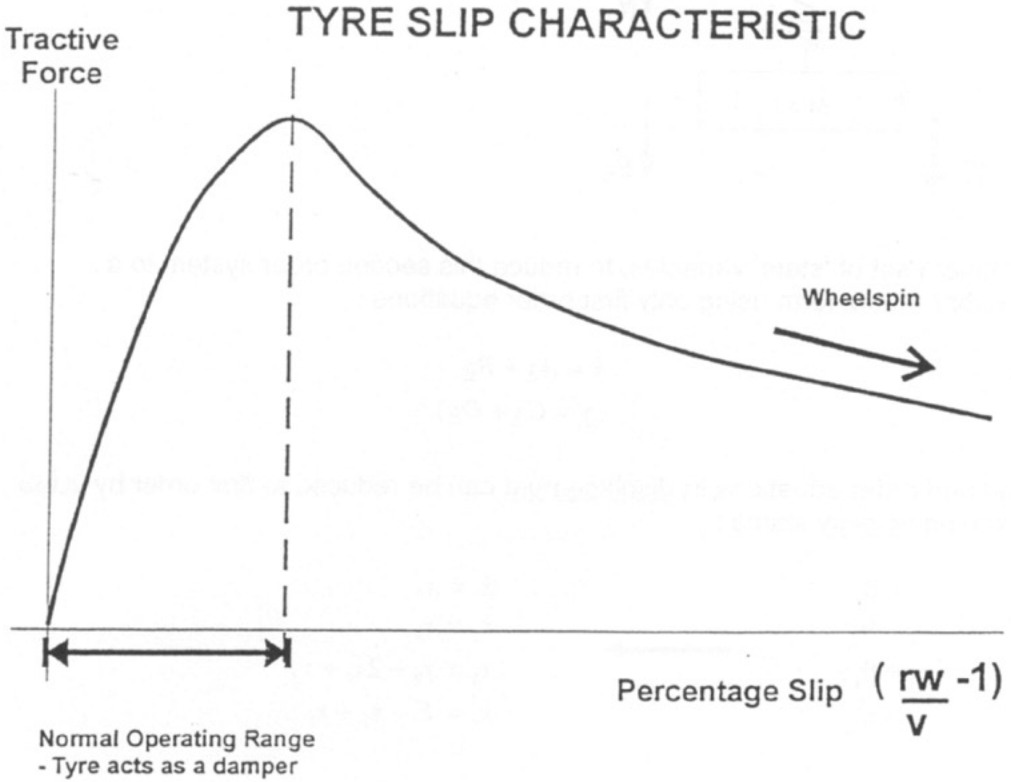





# A Simple Tyre Model

- Longitudinal force in acceleration is generated through contact patch.
- The amount of force generated is proportional to ‘slip’ i.e. velocity difference between tyre (translation) and vehicle (translation).
- Sharply rises, reaches a peak then falls. Stiction vs viscous friction.

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# Vibration Analysis

**Create state matrix**

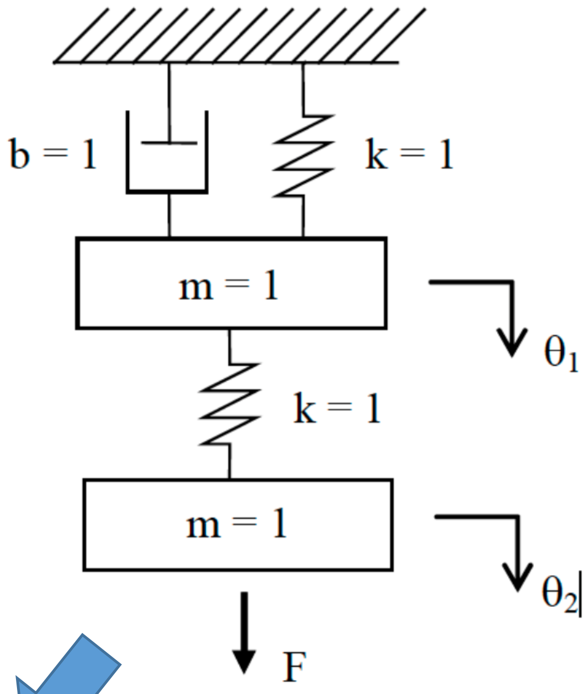
$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ -2 & 1 & -1 & 0\\ 1 & -1 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 0\\ 1\end{bmatrix}\mathbf{F}$$

$$\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u}$$

**Create output matrix**

$$\mathbf{y} = C\mathbf{x} + D\mathbf{u}$$

**Draw schematic**




**Determine system equations**

$$F - l(\theta_2 - \theta_1) = l\ddot{\theta}_2$$

$$l(\theta_2 - \theta_1) - l\theta_1 - l\dot{\theta}_1 = l\ddot{\theta}_1$$

**Define states**

$$x_1 = \theta_1$$
$$x_2 = \theta_2$$
$$x_3 = \dot{\theta}_1$$
$$x_4 = \dot{\theta}_2$$

**Substitute state variables**

$$\dot{x}_1 = x_3$$
$$\dot{x}_2 = x_4$$
$$\dot{x}_3 = x_2 - 2x_1 - x_3$$
$$\dot{x}_4 = F - x_2 + x_1$$

# Vibration Analysis – Modal analysis

Previously
covered
in Section 6

- Remember that;

$$\theta(t) = \mathrm{Re}\left\{\mathbf{u}_1 e^{\lambda_1 t} + \mathbf{u}_2 e^{\lambda_2 t} + \ldots + \mathbf{u}_n e^{\lambda_n t}\right\}$$

- Where each term is;

$$\mathbf{u}_1 e^{\lambda_1 t} = \mathbf{u}_1 e^{(\sigma + j\omega)t} = \mathbf{u}_1 e^{\sigma t} e^{j\omega t} = \mathbf{u}_1 e^{\sigma t}(\cos(\omega t) + i\sin(\omega t))$$

- For a single component;

$$\theta(t) = \mathbf{u}_1 e^{\lambda_1 t}$$ and $$\dot{\theta}(t) = \lambda_1 \mathbf{u}_1 e^{\lambda_1 t}$$

- So that;

$$\mathbf{x}(t) = \mathbf{v}_1 e^{\lambda_1 t}$$ where; $$\mathbf{v}_1 = \begin{bmatrix} \mathbf{u}_1 \\ \lambda_1 \mathbf{u}_1 \end{bmatrix}$$

# Vibration Analysis – Modal analysis

Previously covered in Section 6

- For free vibration;

$$\lambda_1 \mathbf{v}_1 e^{\lambda_1 t} = A \mathbf{v}_1 e^{\lambda_1 t}$$

- So that;

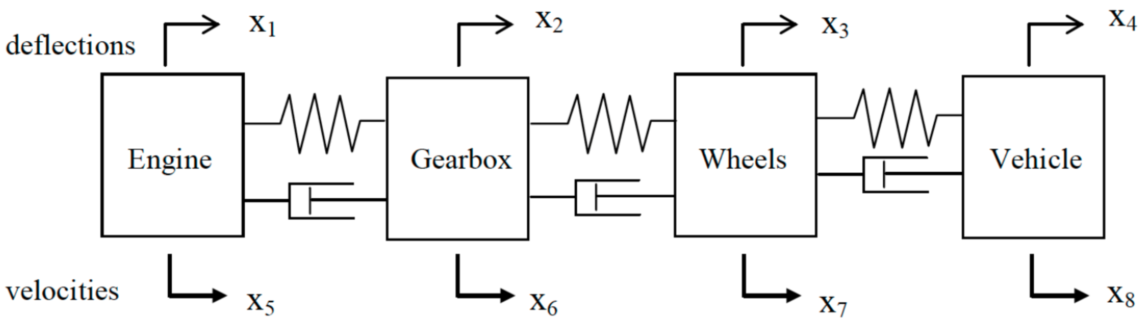
$$\lambda_1 \mathbf{v}_1 = A \mathbf{v}_1$$

- Which means that $\lambda_1$ is a eigenvalue of $A$ and $v_1$ is the corresponding eigenvector – from the definition of what an eigenvalue is.

- For the spring-mass-damper system previously (given parameter values);

$$\lambda_1 = -0.35 + 1.5\mathrm{j}$$
$$\lambda_2 = -0.35 - 1.5\mathrm{j}$$
$$\lambda_3 = -0.15 + 0.63\mathrm{j}$$
$$\lambda_4 = -0.15 - 0.63\mathrm{j}$$

# Eigenstructure – Four mass driveline model




```
[A,B,C,D] = linmod('drive4');
[V,D] = eig(A);
```

…and as if by magic!

| | $\lambda_1 =$ 0 | $\lambda_2 =$ -779.5 | $\lambda_3 =$ -86.9+366.4i | $\lambda_4 =$ -86.9-366.4i |
|---|---|---|---|---|
| x1 | 0 | 0.0000 | -0.0000 - 0.0001i | -0.0000 + 0.0001i |
| x2 | 0 | -0.0000 | 0.0006 + 0.0026i | 0.0006 - 0.0026i |
| x3 | 0 | -0.0013 | 0.0000 - 0.0000i | 0.0000 + 0.0000i |
| x4 | 1.000 | 0.0000 | -0.0000 - 0.0000i | -0.0000 + 0.0000i |
| x5 | 0 | -0.0000 | 0.0269 + 0.0007i | 0.0269 - 0.0007i |
| x6 | 0 | 0.0129 | -0.9996 | -0.9996 |
| x7 | 0 | 0.9998 | 0.0059 + 0.0116i | 0.0059 - 0.0116i |
| x8 | 0 | -0.0150 | 0.0003 - 0.0003i | 0.0003 + 0.0003i |

| | $\lambda_5 =$ -2.40 + 22.26i | $\lambda_6 =$ -2.40- 22.26i | $\lambda_7 =$ -1.4676e-006 | $\lambda_8 =$ 1.4676e-006 |
|---|---|---|---|---|
| x1 | -0.0035 - 0.0329i | -0.0035 - 0.0329i | 0.5000 | -0.5000 |
| x2 | -0.0039 - 0.0284i | -0.0039 - 0.0284i | 0.5000 | -0.5000 |
| x3 | -0.0068 + 0.0040i | -0.0068 + 0.0040i | 0.5000 | -0.5000 |
| x4 | 0.0005 + 0.0038i | 0.0005 + 0.0038i | 0.5000 | -0.5000 |
| x5 | 0.7403 | 0.7403 | -0.0000 | -0.0000 |
| x6 | 0.6412 - 0.0186i | 0.6412 - 0.0186i | -0.0000 | -0.0000 |
| x7 | -0.0720 - 0.1618i | -0.0720 - 0.1618i | -0.0000 | -0.0000 |
| x8 | -0.0848 + 0.0025i | -0.0848 + 0.0025i | -0.0000 | -0.0000 |

# Matlab Example Output

[V,D] = eig(A)

```
D =

   1.0e+02 *

   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i  -5.4924 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i  -0.8696 + 3.6644i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i  -0.8696 - 3.6644i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i  -0.0332 + 0.2219i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i  -0.0332 - 0.2219i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i  -0.0000 + 0.0000i
```

```
V =

   0.0000 + 0.0000i   0.0000 + 0.0000i  -0.0000 - 0.0001i  -0.0000 + 0.0001i  -0.0048 - 0.0322i  -0.0048 + 0.0322i  -0.5000 + 0.0000i  -0.5000 + 0.0000i
   0.0000 + 0.0000i   0.0001 + 0.0000i   0.0006 + 0.0026i   0.0006 - 0.0026i  -0.0053 - 0.0278i  -0.0053 + 0.0278i  -0.5000 + 0.0000i  -0.5000 + 0.0000i
   0.0000 + 0.0000i  -0.0307 + 0.0000i  -0.9995 + 0.0000i  -0.9995 + 0.0000i   0.6340 - 0.0265i   0.6340 + 0.0265i  -0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i  -0.0001 + 0.0000i   0.0269 + 0.0007i   0.0269 - 0.0007i   0.7306 + 0.0000i   0.7306 + 0.0000i  -0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0018 + 0.0000i   0.0000 - 0.0000i   0.0000 + 0.0000i  -0.0096 + 0.0041i  -0.0096 - 0.0041i  -0.5000 + 0.0000i  -0.5000 + 0.0000i
   1.0000 + 0.0000i  -0.0000 + 0.0000i  -0.0000 - 0.0000i  -0.0000 + 0.0000i   0.0007 + 0.0037i   0.0007 - 0.0037i  -0.5000 + 0.0000i  -0.5000 + 0.0000i
   0.0000 + 0.0000i  -0.9994 + 0.0000i   0.0104 + 0.0139i   0.0104 - 0.0139i  -0.0592 - 0.2260i  -0.0592 + 0.2260i  -0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0151 + 0.0000i   0.0002 - 0.0003i   0.0002 + 0.0003i  -0.0839 + 0.0035i  -0.0839 - 0.0035i  -0.0000 + 0.0000i   0.0000 + 0.0000i
```

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# Eigenstructure – Four mass driveline model

- Normalising (as in previous example)
  - Shuffle;

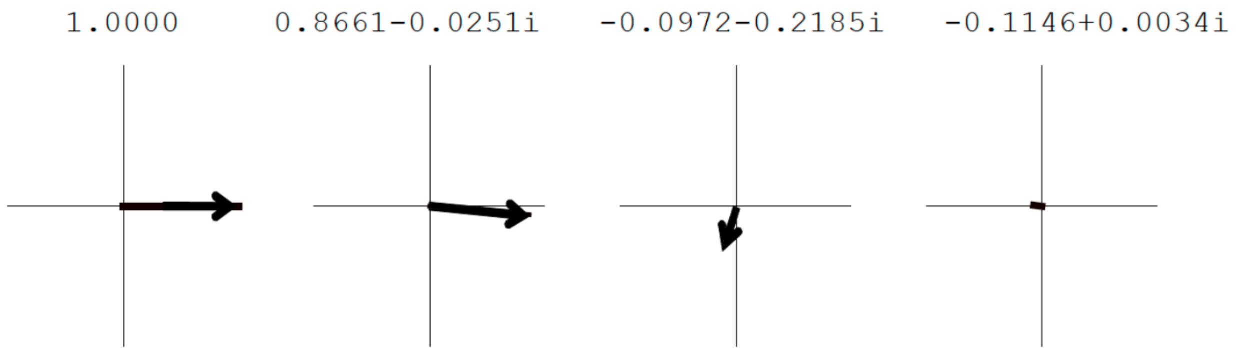
| | | | |
|---|---|---|---|
| 0.7403 | / 0.7403 | = | 1.0000 |
| 0.6412 - 0.0186i | / 0.7403 | = | 0.8661 - 0.0251i |
| -0.0720 - 0.1618i | / 0.7403 | = | -0.0972 - 0.2185i |
| -0.0848 + 0.0025i | / 0.7403 | = | -0.1146 + 0.0034i |

  - Rattle:

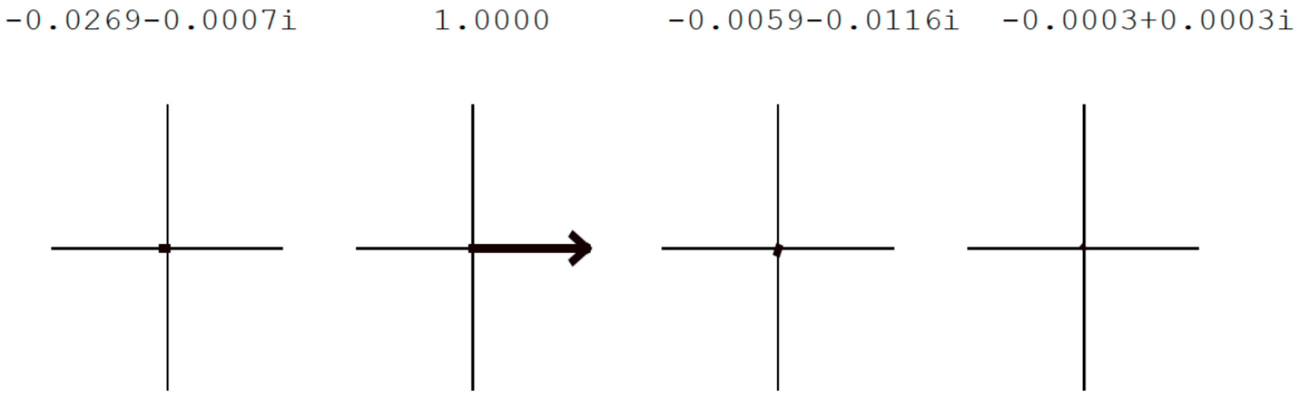
| | | | |
|---|---|---|---|
| 0.0269 + 0.0007i | / -0.9996 | = | -0.0269 - 0.0007i |
| -0.9996 | / -0.9996 | = | 1.0000 |
| 0.0059 + 0.0116i | / -0.9996 | = | -0.0059 - 0.0116i |
| 0.0003 - 0.0003i | / -0.9996 | = | -0.0003 + 0.0003i |

# Eigenstructure – Four mass driveline model

- And plotting
  - Shuffle;



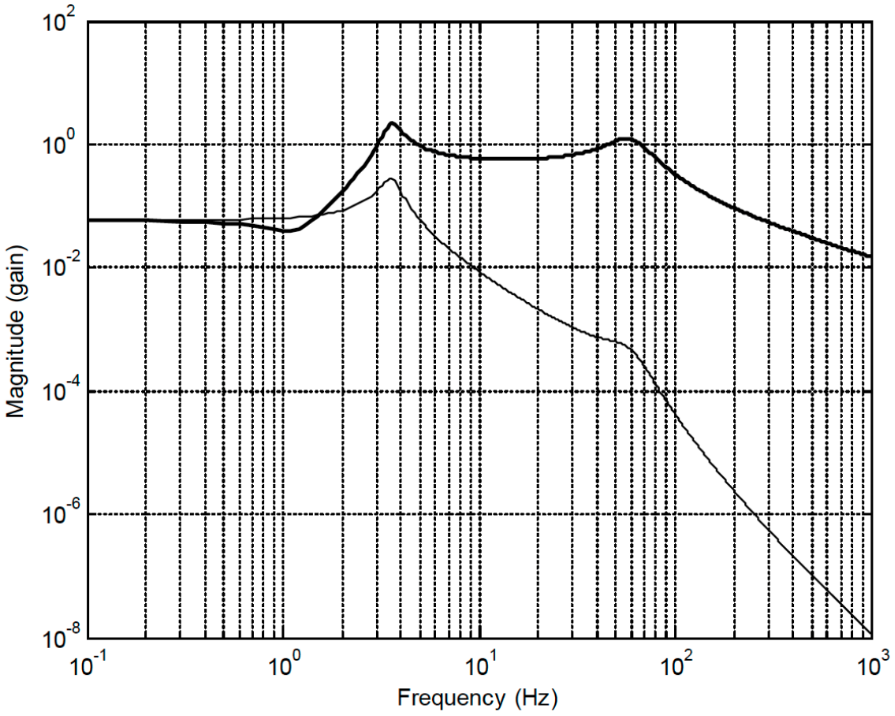

  - Rattle;



# Eigenstructure – Four mass driveline model

- Bode plot to show frequency response between input torque, vehicle and transmission acceleration.

```
[A,B,C,D] = linmod('drive4');
sys = ss(A,B,C,D);
f = [0.1:0.1:1000]';
[mag,phase] = bode(sys,f*2*pi);
mag = squeeze(mag)';
loglog(f,mag);
grid on;
```

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